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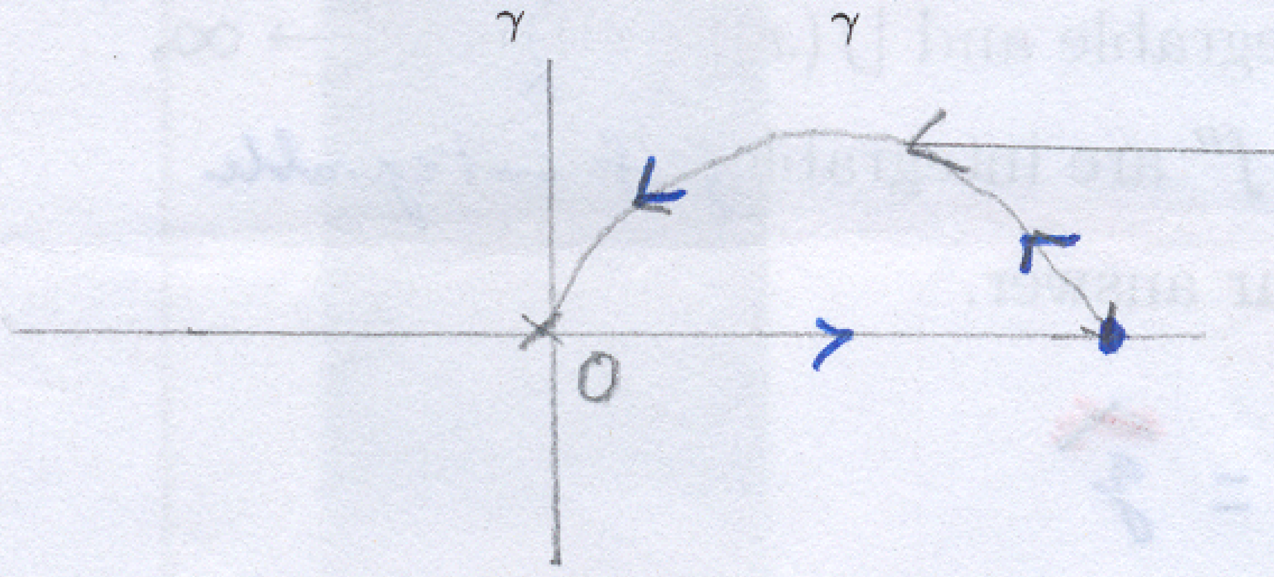
Indian Statistical Institute
First Semester Examination 2004-2005
B.Math II Year
Analysis III

Time: 3 hrs Date: 22-11-2004

Answer all questions. The maximum you can score is 60. You may bring your classroom notes or any other material I have distributed in class. However textbooks are not allowed in the exam. You may use theorems proved in the class, but state them CLEARLY. ALL ANSWERS MUST BE PROPERLY JUSTIFIED.

1. The curve γ is defined by : $\gamma(t) = (t, \sin t)$, $0 \leq t \leq \pi$. Check if ~~Prove that~~ $\ell(\gamma)$, the length of γ satisfies the inequality.
$$\ell(\gamma) \geq (\sqrt{2}+1)\frac{\pi}{2}.$$
[5]

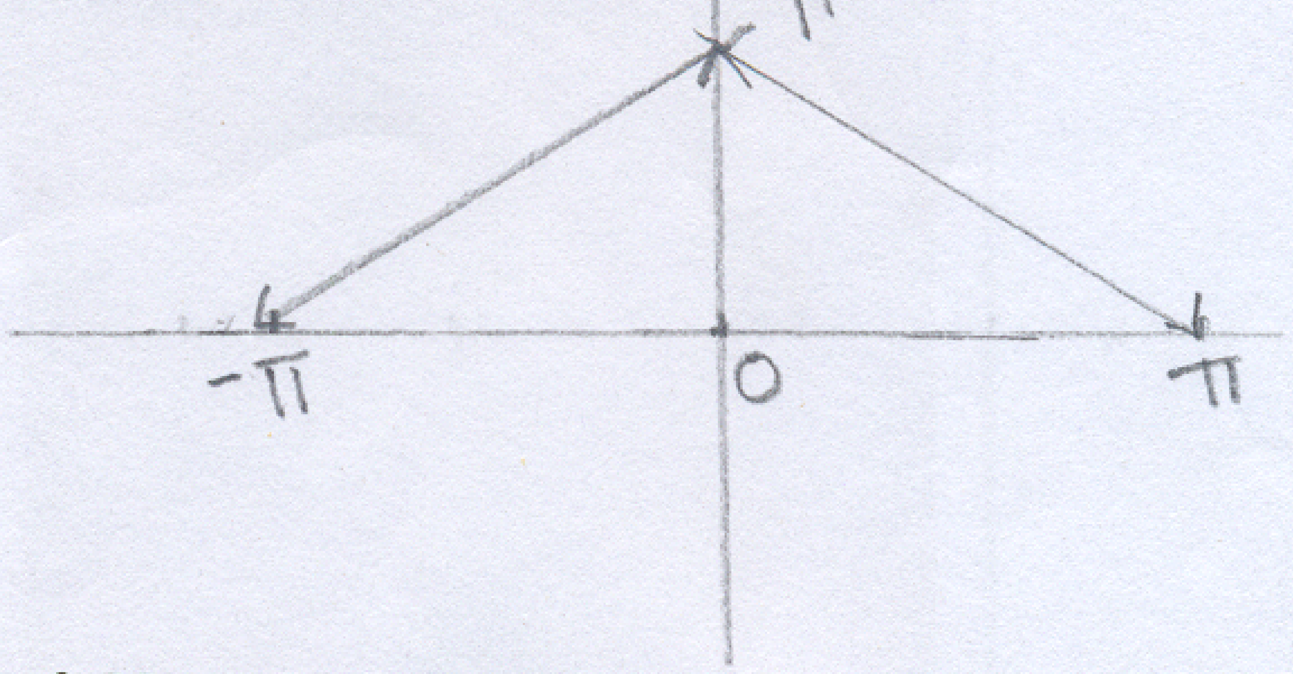
2. Let γ be the curve shown below. Let $F = (F_1, F_2)$ be given by $F(x, y) = (y, 0)$. Evaluate $\int\limits_\gamma \vec{F} \cdot \vec{ds} = \int\limits_\gamma (F_1\, dx + F_2\, dy)$. [5]



Graph of $y = \sin^2 x$

3. Locate the zeros and poles of the function $f(z) = \frac{z-1}{z^2+z-2}$. What are the orders of these zeros and poles? [5]

4. Find the Fourier series of the 2π-periodic function f whose graph between $-\pi$ and π is shown below: [15]



Find a value of N such that
$$\int_{-\pi}^{\pi} |s_N^f(x) - f(x)|^2 \leq \frac{1}{1000}.$$

5. Consider $\sum\limits_{n=-\infty}^{\infty} a_n e^{inx}$ where $a_n = 0$ if $n \le 4$ and $a_n = \frac{1}{\sqrt{n \log n}}$, $n > 4$. Can the series above be the Fourier series of a 2π- periodic piecewise continuous function? Justify your answer. [10]

6. Let $f(x) = \sum\limits_{n=2}^{\infty} (a_n)^x$ where $\cancel{1}\,0 < a_n \le \frac{1}{n}$, $n \ge 2$. Prove that this defines a continuous function on $(1, \infty)$ by showing that the series converges uniformly in every interval of the form $[\delta, \infty)$, where $\delta > 1$. Prove that f is a differentiable function in $(1, \infty)$. [10]

7. Let f be defined on the real line by $f(x) = x$, for $-\pi \le x \le \pi$ and $f \equiv 0$ outside this interval. Compute the Fourier transform of this function. [5]

8. If f is as above, Compute $(f * f)(x)$. What is $(f * f)^\wedge(\lambda)$? Let $g(x) = \sum\limits_{n=-\infty}^{\infty} f(x + 2\pi n)$. Determine g. [10]

9. Let $g(x) = \frac{1}{1+x^2+x^2 \sin^2 x}$. Does there exist a function f defined on $I\!R$ with the following properties?

i) f is twice continuously differentiable,

ii) f is integrable and $|f(x)| \to 0$, $f' \to 0$ and $f'' \to 0$ as $|x| \to \infty$,

iii) f' and f'' are integrable, f integrable

and $\hat{f} = g$

Justify your answer. [5]